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WHAT MATHEMATICAL SUBJECTS SHOULD BE INCLUDED IN THE COLLEGE CURRICULUM?

By F. J. Holder.

In the forenoon of this twentieth century, when democracy is asserting itself, no less in the political realm than against the old aristocracy of learning, we who are voluntarily yoked to the common load of teaching mathematics must realize its present state of unrest. Possibly this is not more noticeable in mathematics than in many other branches of the curriculum, and it is probably a bit less conspicuous in the colleges and universities than in the secondary schools; and furthermore, this unsettled condition is by no means confined within the walls of our American institutions, but its constant throbbing is felt in the educational pulse of every progressive country in the world. There seems to be an ever-present desire for a change without first counting the cost of the move; a mere effort to have things different, with no well-defined plan of having them better.

There are a few among us, and many in France, who advocate the substitution of a geometry of motion for Legendre's geometry of congruence, without furnishing sufficient evidence to convince us that the former is not more abstract and ill arranged than the latter. England has recently attempted the elimination of Euclid, with the reward of a temporary mathematical condition that can hardly be satisfactory to any one, and certainly her outfielders have not taken kindly to the play. In many of our private, and not a few state, institutions this disturbance has assumed the form of petty mathematics, ill-arranged courses, or the hope that, except in very special cases, mathematics may be erased altogether from the *required* list of studies prescribed in the college curriculum. Prominent among those entertaining this last fond hope may be mentioned the name of Harvard, which has recently made a forward pass in this direction, and there seems to be considerable argument as to whether or not she has landed safely and is entitled to the lead. Some of us believe there was a fumble involved and prefer to withhold our applause until after the umpire's decision, which in this case may be given out somewhat later in the game. And so it is here and there and everywhere, we find this longing for something better, but often with the result that something worse appears. I am told that it is not an uncommon occurrence for a discontented child to cry for water when, in reality, the troubled sea needs oil.

I believe it is the duty of the colleges to serve the commonwealth, and many have seen fit to lower, temporarily at least, the general standard in order to meet the needs of the great mass of boys and girls who are coming in from the various high schools and academies, in place of the more selected and, therefore, talented lot which formerly entered. This is sometimes interpreted to mean that we must take off the rich cream which requires sustained effort and logical reasoning and pour out the skimmed milk, subsequently diluted to the proper weakness, and invite the student to drink thereof and grow and wax strong.

Some claim that we should have only such mathematics as is immediately practical, the power and resourcefulness of the potentially practical being lost sight of in the desire for the present need, but I believe that investigation will show that this demand comes largely from technical schools and vocational schools of one kind or another.

Finally there are those who go so far as to assert that mathematics should not appear as a science at all, but that when a real problem arises its solution should be effected and no other problems should be given. Doubtless the adherents of this last group are firm believers in the hand of Providence, and any arguments admissible to a place in this short paper would prove futile and powerless to shake their faith.

As a marked contrast to some of the modern conceptions of mathematics as a science, it might be of interest to give the following quotation ascribed to Novalis (1772–1801) who was *not* a *mathematician* but a German romantic *author,* once of cosmopolitan renown. Carlyle delighted to read and re-read his novels and recommended the author to the world as a clear thinker. Novalis wrote: "The life of the Gods is mathematics. All divine messengers must be mathematicians. Pure mathematics is religion. Mathematicians are the only fortunate ones. The mathematician is naturally an enthusiast. Without enthusiasm, no mathematics."

I freely admit that I do not fully agree with any one of the above-named views for, although I do find a great deal of satisfaction and entertainment in figures at times, I cannot conscientiously bow down and worship the subject as did Novalis. And again, when I observe our modern specimens of model mathematicians who are the authors of our magazine articles and our most familiar text-books, I sometimes find the long hair which is so characteristic of the species, but the long symbolic wings one naturally expects to see upon the shoulders of a divine messenger are not always so much in evidence, and to all outward appearances these Novalis deities have somewhat the semblance of men.

On the other hand, I am not inclined to say that mathematics is intended for its immediate application only; far from it; one must love it for its own sake; but the very fact that it has done so much to shed light upon the laws of nature and to explain the immensity and purpose of the universe and that it is recognized as a power in the removal of the old ideas of idolatry and superstition from among us serves as a mighty shield to protect us from the darts of the skeptic.

The strictly practical men remind us that many of our Carnegies, Morgans, and Rockefellers, though ardent admirers of figures, are not great mathematicians, and they also refer with much pride to prominent discoverors and inventors who have accomplished similar achievements. To such argument, Henri Poincare once replied: "As it is often partially true; bold undertakings are frequently due to those who are free from dizziness, and to prevent dizziness one must not see too clearly. Of these adventurers, only the successful are counted, not those who break their necks." Not the less true is it that modern industrial development, considered as a whole, would have been impossible but for the advancement of science, and to no small degree that of mathematics, which gives form to the dreams of the superficial and renders possible the ideals of the visionary.

In the college curriculum at the present time, we might not be able to require more than one year's work in mathematics, even if we were inclined to try, and the average freshman, in the three, four, or possibly five, hours per week allotted to this subject, can not hope to do more than master the fundamental principles of algebra, geometry, and trigonometry, so long as he comes to us with the limited preparation now prescribed for entrance to college. It might be well to mention just here that Professor Höfler, of the University of Vienna, advocates the introduction, into the secondary schools, of courses of mathematical instruction which have to do with modern questions of values, for example, the question of the function concept, of " functionally thinking " in mathematics. The late Professor Tannery first raised this question for such schools in France some time ago; then Professor Klein took it up and gave it a definite standing in Germany and Austria, and, while I do not wish to alarm those of us present who are members of our efficient body of high school teachers, I might say that Professor David Eugene Smith, of Columbia University, predicts that we shall be hearing a great deal of this question in the next ten years in the secondary schools of America. I believe the teacher of high school mathematics should be perfectly familiar with such topics as irrational and imaginary numbers, algebraic and transcendent numbers, the relation of mathematics to physics, and the introduction to the calculus. He should have some conception of the significance of number, of space, of intuition in mathematical teaching, and of fundamental concepts, the relation of logic to mathematical proof, and the interesting yet not sufficiently appreciated question of "mathematical sophisms."

In a report submitted on January 8, 1911, to the International Commission on the Teaching of Mathematics, by Committee XII., Subcommittee 1, composed of: Chairman, C. B. Upton, Teachers College, Columbia University; R. D. Bohannan, Indiana University; L. G. Weld, University of Iowa; C. D. Rice, University of Texas; and A. L. Candy, University of Nebraska, it is stated that "The *ideal* preparation for teaching mathematics in the secondary schools and the first two years of college should approach the completeness of the preparation in France and Germany, where one must have a state license, obtained only by passing a most rigid examination, before he can hope to do anything worth while in secondary teaching." In this report, we also find the following prophecy: "As the public is now beginning to recognize that teaching is a profession, a feeling which will undoubtedly increase as the years go by, the time will undoubtedly come when secondary teaching will be sufficiently attractive financially to enable us to demand from the prospective teacher some such preparation as the following. On the side of pure mathematics, we may expect the calculus, differential equations, solid analytic geometry, projective geometry, theory of equations, theory of functions, theory of curves and surfaces, theory of numbers, and some group theory. On the applied side, we should demand a strong course in mechanics, theoretical and practical astronomy, descriptive geometry, and some mathematical physics with a thorough course in experimental physics. To this should be added special courses on surveying and general applications of mathematics that the student may see to what all of the above work is leading. As pedagogical training, there should be included a strong course on the teaching of secondary mathematics with observation and practice teaching under expert supervision, a course on the history of mathematics, at least one graduate course on the history and teaching of mathematics, and a course of an encyclopedic nature dealing critically with the field of elementary mathematics from the higher standpoint. Such an *ideal* preparation, with the exception of about half of the pedagogic training outlined, is no more excessive and severe than the requirements in France today for the secondary teaching license known as the *agregation,* and we can demand this if the public will give teaching the recognition it deserves.

As I have said before, I believe that the college should serve the commonwealth. By this I do not mean that she should cater to the whims of every individual in the community nor would I intimate that she should attempt to perform all of the educational duties of the city or state in which she is located. It is not my purpose to discredit the work of the technical schools and professional schools of one kind or another, for I am convinced that they have a great mission to fulfill, and many of them are meeting their obligations admirably. It is thought by some that it is the business of the college to furnish executives and teachers for the numerous secondary schools and institutions of higher learning but, while I am proud of the important part she has always played, and will continue to play, in this rôle of honor, I believe she has a broader field of activity in which to cultivate and harvest her growing products. It is one of her noble functions to make men and not machines. It is true, however, that the college is the foundation or center upon which, or around which, the graduate school and professional schools of the university are built and, while many universities have tried to exist without this factor prominent in their midst, they are coming to realize more and more every year that success or failure, to a greater or less extent, hangs in the balance of this potent factor, and one by one they are installing such a department where it did not already exist.

It is difficult for me to conceive of a college without its department of English and mathematics with their courses (prescribed or elective as the case may be) adorning the curriculum by their presence here and there in each of the four years' work scheduled. When I name certain subjects in pure or applied mathematics with which the ideal teacher of secondary mathematics should be familiar, I believe the college should, in so far as it is possible, provide him with the opportunity to become familiar with such subjects either by offering the individual courses desired or by presenting the ones she does offer in such a way as to give him a thorough knowledge of the fundamental principles underlying the whole scheme and to launch him safely upon the high sea fully equipped with glass and compass and the determination to search out the truth. And when he has found the truth, his preparation should enable him to recognize it, to read it intelligently, to assimilate it, and thus to become master of those things he is supposed to conquer.

It might appear that I am advocating the arrangement of the college curriculum specifically to meet the needs of the secondary school teacher only and am forgetting the large majority of college graduates who do not enter the teaching profession, and immediately the question arises in the minds of those of you who may be intimately connected with well-established teachers' colleges or training schools: "Why don't the colleges refuse admittance to those men and send them to the teachers' colleges where they are fully prepared and equipped to offer this ideal training?" A partial answer to this question is the fact that colleges are more numerous than teachers' training schools, and many there are who find it possible to avail themselves of the opportunities offered by a near-by college but for whom it would be extremely inconvenient, to say the least, to spend the same time away from home.

All students who seek college training, whether required to take mathematics for one, two, or more years, or left to elect it in the wide open elective system, should not only be given the opportunity of taking such subjects as have been mentioned and others of a similar nature which may appeal to their individual tastes, but they should be brought to realize the "values" in mathematical study. Here the question arises: "What are the 'Values' in mathematical study? What is there in the subject to justify its place in the college curriculum?" I believe the answer should be broad and comprehensive. We should recognize the importance of mathematics as a preparation for the study of other subjects and because of the use we can make of it, that is, for its practical applications; but we should also recognize, as the *underlying reasons* for its study, the seeking of mathematical truths for their own sake, that is, for culture; and for the sake of the training acquired in its study, that is, for mental discipline. By mental discipline, I mean a training in intuition, judgment, memory, imagination, reasoning powers; an improvement in ability to concentrate, to think clearly, accurately, and logically; to recognize the essential elements in a problem, to note relationships, apply principles, and understand cause and effect. I believe that general abilities are gained through the exercise of the mind on a special subject, especially mathematics; in other words, I believe that the power, the sense of mastery, the standards and the ideals acquired through mathematical study are carried over into other subjects and the general activities of life. Were we deprived of the practical aspects of the subject entirely, I should consider the cultural and disciplinary values as necessary and sufficient to justify me in my profession. The modern ideal, however, lies between the two extremes of the strictly logical mathematics of the Greeks and the strictly practical mathematics of the Romans and, while we sometimes insist on the formal study of geometry and algebra, we use all the modern devices possible for presenting the subject and making it clear and vital. A certain standard of logical rigor is not inconsistent with the use of intuition, concrete exer-